Panel T01P10 Session 1

*Systems Theory and Modelling for Public Policy: System Dynamics,
Agent-based Models, and Other Approaches*

Title of the paper

*Game theoretic study on methods for measuring costs of decision-
making and effects of consensus building*

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Abstract

In the process of decision-making, "how to decide" is an issue as important as "what to decide," but little attention has been paid to it. Our study concentrates on "how to decide" and originally devise a method to measure "cost of decision-making" and the degree of consensus in a conference. We analyze decision-making situations at the conferences by the game theoretical framework called "simple games." Our new approach may become a unified way to assess the degree of consensus at all decision-making organizations, and our study suggests the possibility to apply "consensus building" rule to Congress.

Keywords: Democracy; Collective decision-making; Consensus building; Decision rule; Game theory; Simple games; Congress

1. Introduction

In the present era, the government's policy is extremely important. So far, many previous studies in economics address efficiency of resource allocation in market mechanisms and show that the maximization of individual interest consequently maximizes the social welfare. On the other hand, other studies show that the market mechanisms could lead to some undesirable results such as e.g., "market failure," and "the tragedy of the commons," etc. In recent years, it is well known that the government also plays a more and more important role in preventing the above problems. Because the decision-making of the government is denoted by an implementation of "policies," the policies are indispensable for maintaining the social welfare.

Unfortunately, the government's policy is not always made and implemented perfectly. In many social problems such as poverty, inequality, pollution and conflict, that threaten human dignity and security, it is apparent that the government is not always able to protect the society from the social problems. Additionally, the public even sometimes suffers a loss due to the intervention of the government although the initial purpose of the intervention is to protect the

public interests. In other words, there is always a possibility that the government will cause "government failures." Therefore, when politics is being managed by democracy that is easily influenced by people's will, particular attention is needed to help the government to avoid "government failures."

Policies for democracy, that is, improvements and expansion of democracy, which are regarded as political management methods, have not received sufficient attention yet. In the history of repeated disastrous wars and conflicts, democracy is thought of as one of the best political management methods. Along with the spread of democracy, people further recognize the existence of diversity in many aspects of the society. Nevertheless, there are many problems such as conflict and division due to mutual intolerance among people, and there still exist problems such as confrontation and division resulting in many difficulties in several countries. Moreover, our society is still immature in the sense that some concepts such as diversity and tolerance are still not incorporated into social institutions. In respect of politics under a democratic regime, the issue that how to decide is as important as the issue that what is decided, because the correctness of policies and the smooth implementation of policies are not always achieved simultaneously.

In this paper, we aim to propose a practical methodology to analyze the situation of decision -making, to promote smooth policy implementation. Under the present democratic regime, we strictly distinguish between politics and policy, and examine the smooth implementation of policies by proposing practical methodologies as public policy. Our study can be rephrased as "policy in the political field." The methodology of the present paper is on the basis of the theory of public choice. Public choices present rational behavior in politics or democracy by mathematically analyzing decision-making. Rational behavior should be applied in politics. However, knowledge of public choices accumulated by the efforts of many pioneers has not been fully utilized. Therefore, our interest is the application of the theory of public choice. We are concerned with an improvement of social decision-making system and reconsider the concept of democracy.

At the present stage, we will focus on determining of policy, that is, how to decide under a democratic regime. More simply and essentially speaking, it is a matter of "consensus." Under a democratic regime, the decisions necessary for society are dependent on people's will, and it is called collective decision-making in public choice. The people's will stand for a preference of all individuals. Therefore, the essence of collective decision-makings is to aggregate the preferences of many individuals optimally and make decisions convince society. There will be no objection to the fact that politics is responsible for such activities. In respect of politics under a democratic regime, the issue on how to decide should be of the same interest as the issue of what is decided. However, much attention has not been paid to the issue that how policies can be decided, compared to the fact that many policies are created each year.

Let us take an example of policy decision and think about this problem. Policies related to public goods are the most appropriate examples of important issues that politics should decide. Studies on the supply of public goods (e.g., Breton 1998) point out the importance of the unanimous decision on the optimal quantity of public goods. Decision-making methods based on majority rule are widely used at an actual political scene such as the conference in Congress. The discrepancy between these two decision-making methods has not been regarded as a serious problem in the past. Under general parliamentary politics, people opposing the majority are deemed to have agreed to the final decision, no matter whether they agree with the decision or not. The majority rule is considered to be the most reasonable and best way for our society, which is often difficult to decide unanimously in many cases. It is still uncomfortable to fully identify the consensus by "majority" rule and the consensus by "unanimity" rule. Such feeling of discomfort to common sense is the fundamental question of this study. Consequently, the issue on how to decide is equivalent to the issue on how to adjust the ideal and the reality about "consensus" among people.

The purpose of the study is to develop a method to measure the degree of consensus at real conferences. Our main concern is the issue about how to decide in politics under democracy, (or equivalently how to decide in the consensus in politics). To tackle the issue on how consensus

should be achieved in the conference of all organizations including Congress and other public committees, we first observe the state of consensus at the conference. It is important to be able to grasp to what extent the majority rule has achieved consensus, or how serious the conflict becomes. To deal with this problem, we employ the mathematical method called "simple games" in game theory. "Simple games" is the most suitable approach to derive measurable numerical values from ambiguous circumstances such as "decision-making" and "consensus."

Our method puts the theory of public choice into practical use by applying the game theoretical framework. The method is based on the theory of "cost of decision-making" presented by Buchanan and Tullock (1962), the pioneer study of public choice. By analyzing collective decision-making in terms of cost, they suggest that decision makers should decide at the equilibrium point where their costs are minimized. However, due to the difficulty of measuring "cost of decision-making" that possibly changes for each proposal, there are few studies concentrating on how to utilize their theory in actual decision-making situation such as conferences. Moreover, since decision makers' preference for the proposal varies with the situation, it is always difficult to determine the optimum majority rule. The feature of this study is that in order to avoid such obstacles, the preference is derived from the meta-situation of decision-making rather than the proposal. Employing this method, we find it is possible to set each preference of decision makers for each proposal in simple form, which makes it convenient to measure the degree of consensus at the conference with a uniform standard. Our new approach applying the game theoretical framework may become a unified way to evaluate the degree of consensus at all decision-making organizations, including, e.g., Congress and other public committees.

The rest of our paper is organized as follows: Chapter 2 looks back to previous major relevant literature and unresolved problems; Chapter 3 introduces framework and method employed in present study; Chapter 4 models a simple Congress under the game theoretical framework and gives analyses on the model; We discuss the result of our model in Chapter 5; Chapter 6 gives conclusion.

2. Literature Review

Public choice is making a great contribution by using many mathematical frameworks for problems concerning decision-making or consensus. In this chapter, we look back to the previous researches in the theory of public choice and discus some relevant unresolved problems. Let us first turn to the study by Buchanan and Turlock (1962). They study "consensus" from the viewpoint of "cost of decision-making."

Buchanan and Turlock (1962) categorizes the cost required by decision-making into two types, "decision-making costs" and "external costs." "Decision-making costs" is a cost expected as a result of an individual participating in an organizational activity. For example, it is a cost of negotiations and adjustments to try to pass through the proposals he or she supports. They explain as follows.

> "If two or more persons are required to agree on a single decision, time and effort of another sort is introduced—that which is required to secure agreement. Moreover, these costs will increase as the size of the group required to agree increases. As a collective decision-making rule is changed to include a larger and larger proportion of the total group, these costs may increase at an increasing rate." (p.68)

In other words, this cost can be regarded as the cost required to overcome the conflict situation within the group and reach a consensual situation.

Meanwhile, "external costs" is the cost that individuals accept as a result of acts of others who do not have direct control. Participating in decision, when the introduction of undesirable taxation for a decision maker is decided by collective decision-making, it is a loss and creates dissatisfaction for him or her. The "unanimity" rule is the only collective decision-making rule not to cause such "dissatisfaction."

With these concepts, they illustrate quite ambiguous situation of decision-making and also presents a reason for adopting a majority rule in democracy. Since "decision-making costs" and

"external costs" have a trade-off relationship similar to the one between "supply" and "demand," it is desirable to make decisions at the equilibrium point where the sum of both costs is minimized. More strictly speaking, their study suggests the optimal K-majority decision rule that is most desirable for society. K is the threshold of "aye" votes that are necessary to pass proposals at an n-person conferences. The threshold varies with the case that the proposal passes by only one voter (K =1) to the case where all voters' approval is required (K= n). They argue that it is optimal to make decisions at K that minimizes the sum of "decision-making costs" and "external costs." In addition, they demonstrate the efficiency of adopting majority rule in collective decision-making.

Inspired by a classic and persuasive theory proposed by Buchanan and Tulloch (1962), many efforts have been made to reinforce and refine their theory in recent years. Spindler (1990) argues that "the cost of rent seeking" should be considered when deriving the optimum majority rule. "Rent" is the excess profit resulting from regulation (or mitigation) of supply, and "rent seeking" is an activity to secure such excess profit (Krueger (1974)). By analyzing the expected utility, Guttman (1998) shows that the simple majority rule is socially optimal, and reinforces the importance of the theory of "cost of decision-making." Adopting the Buchanan and Tullocks' frameworks, Brennan and Hamlin (2000) points out that it is more important to find an appropriate representative proportion than to lead an appropriate K-majority rule. Mueller (2003) suggests that there is a discontinuity in the curve of the decision-making costs. They explain that if this discontinuity is sufficiently large, the majority rule will be appropriate. Dougherty and Edward (2004; 2011) reconsiders the shape of the decision-making costs curve and the external costs curve, and the authors argue that "external costs" would be almost zero at that stage, as the possibility of decisions decreases as the number of people required for majority voting increases. In other words, they question the theory that the unanimity rule uniquely minimizes "external costs" and try to make the model closer to the real situation. Additionally, Dougherty et al. (2015) and Dougherty and Ragan (2016) expand the study of Dougherty and Edward (2004; 2011) by introducing the viewpoint of "gain," "loss" or "expected utility" when the proposal is passed.

It is needed to note that a majority rule is not the only best decision rule in democracy. The following major studies dealing with voting discuss some problems caused by majority rule: Black (1958) develops "single-peaked preference," and the author formulates the basis for mathematical analysis of democracy. Arrow (1963) proves "general possibility theorem (Arrow's impossibility theorem)." He shows the difficulty of designing collective decision-making rules including majority voting. Sen (1996) proves "value restriction theorem." By using this concept, he expresses partial consensus among voters and extends Arrow's theorem. Campbell and Tullock (1965) concentrates on "probability of voting paradoxes." Riker (1962) studies "minimum winning coalition" and emphasizes that it is needed to focus on the power of decision makers in voting. Brams and Fishburn (1978) presents "approval voting" and shows that there are various forms of voting. These prominent studies reveal the complex nature of the majority rule and imply that the majority rule is not the only decision rule. It is necessary to understand the position of Buchanan and Tullock's study in these series of studies. In other words, from the viewpoint of "cost of decision-making," the majority rule is valid.

Certainly, it is convincing to classify the costs of decision-making into "decision-making costs" and "external costs." However, the following problems should be considered: the shape of these cost curves fluctuates according to the distribution of decision makers' preferences. Van den Doel (1993) deals with this problem, and it concludes that the external costs curve will soar in a case of enacting the Constitution or revising it. We can understand the concept of decision cost. However, the cost curve is not always constant, and its shape is determined dependent on the agenda. To derive optimum results based on the theory of "cost of decision-making," it is needed to know the shape of the cost curve for each agenda. In addition, at the conference, multiple proposals may be submitted. In that case, to clarify the shape of the cost curve for each proposal is a difficult request. The diversity and complexity of the decision-making situation hinder the practice of the theory. Even if it is possible to derive a cost curve, the current situation and people's preferences may always change. Therefore, it is impossible to predict our future preferences. Decision-making is also dominated by the uncertainty of the situation.

3. Framework and Methods

3.1. Concept

This chapter shows a framework for mathematical analysis and a method for measuring the degree of consensus in the conference. The purpose of the study is to propose a general method that can grasp the nature of conflict and consensus at the conference by measuring "decision-making costs" in collective decision-making. We adopt the mathematical model called "simple games" in game theory to derive measurable numerical values from ambiguous circumstances such as decision-making and consensus.

The rough flow of the method proposed by this study is as follows. We first analyze the conference using the game theoretical framework called "simple games" introduced by previous studies (e.g., Peleg (1984); Ramamurthy (1990); Taylor and Zwicker (1999); Inohara (2007); Maaser (2010)). Subsequently, we assume "a conference to determine decision rule for the main conference that is held to make a decision." Assuming a meta-conference, we define a point function to derive the "degree of satisfaction" of each decision maker for each decision rule in such meta-conference. At the same time, by comparing "degree of satisfaction" among decision makers, we derive "degree of dissatisfaction" of decision makers who suffer "external costs." "Degree of satisfaction" and "degree of dissatisfaction" correspond to "decision-making costs" and "external costs" stated in the previous study. "Decision-making costs" stands for "cost of decision-making" as well as the reward for society. However, when "external costs" is high, remarkable conflict possibly arises, which may, in turn, spoils social rewards. On the basis of this fact, it is possible to measure the degree of consensus at the conference.

Furthermore, we apply the concept called "permission" (Yamazaki et al. (2000)) introduced in the previous study to the above meta-conference. Here we define term "permission" to indicate the flexibility of preference. By introducing it to meta-conference, we show that the range of preference of each decision maker is expanded. As a result, it shows that the proposal is uniquely determined in some cases. This situation is called "consensus building," and it reveals changes in

"degree of satisfaction" and "degree of dissatisfaction" by adjusting the point function. Then we measure the effect of "consensus building" by comparing with the degree of consensus before "permission" is applied.

Our method is based on the important theory of "cost of decision-making" (Buchanan and Turlock (1962)). However, the method of measuring the cost has not been sufficiently studied yet. This study sheds light on this problem. We propose a method to measure "cost of decision-making" in the conference and use it to clarify the nature of conflicts and consensus. The feature of our method is to derive "cost of decision-making" from the meta-situation of decision-making, rather than from proposals. Using this method, it is possible to set each preference of decision makers for each proposal without complexity, and further, we are also able to avoid uncertainty depending on the fluid situation.

In this study, we examine the situation where there are three decision makers and three alternative proposals. When the number of decision makers and the number of alternative proposals increases, the situation becomes complicated, and analysis becomes extremely difficult. At this stage, we propose frameworks and methods to conduct the most fundamental analyses.

3.2. Framework

In this study, we employ "simple games" approach to model the decision group originally developed by von Neumann and Morgenstern (1944). This approach is widely applied in cooperative game theory. The framework of the present model is based on previous studies regarding "simple games." In particular, we adopt the framework of Inohara (2007). The framework of the present model is based on the contribution of the previous study, and our main objective is to propose how to utilize the framework.

First, combinations of the decision makers and the proposal are defined as "groups."

Definition 1 (groups). A group C is a pair (N, A), where N is the finite set of all decision makers (DMs), and A is the finite set of all alternatives.

Groups with a list of preferences of DMs build "meetings" for decision-making.

Definition 2 (meetings). A meeting C_R is a 3-tuple (N, A, R), where $C = (N, A)$ is a group, and R is a list $(R_i)_{i \in N}$ of preferences R_i of each DM $i \in N$ on A.

The set of all the linear orderings on A is denoted by $L(A)$. In $R \in L(A)$, the most preferred proposal is denoted by *maxR*. Following Inohara (2007), we give the subsequent notation of preference:

For $i \in N$, the preference R_i of DM $i \in N$ is a linear ordering on A, that is, R_i satisfies the following four conditions: (i) reflexivity: for $x \in A$, xR_ix, (ii) transitivity: for $x, y, z \in A$, if xR_iy and yR_iz, then xR_iz, (iii) anti-symmetry: for $x, y \in A$, if xR_iy and yR_ix, then $x = y$, and (iv) completeness: for $x, y \in A$, xR_iy or yR_ix. The set of all linear orderings on A is denoted by $L(A)$.

Moreover, for $x, y \in A$, xR_iy means that DM i prefers alternative x to alternative y, or is indifferent between alternative x and alternative y. xI_iy means that xR_iy and yR_ix, that is, DM i is indifferent between state x and state y. By the assumption of anti-symmetry, xI_iy implies $x = y$. xP_iy means that xR_iy and "not (yR_ix)," that is, DM i strictly prefers state x to state y.

For meetings, rules for decision-making are necessary and thereby the following "simple games" are employed to express meetings with rules.

Definition 3 (simple games). *A* pair (N, W) of N and a class W of subsets of N is said to be a simple game on N, if it satisfies the following two conditions: (i) effectiveness: $\varnothing \notin W$ and $N \in W$, and (ii) monotonicity: for $S, T \subseteq N$, if $S \in W$ and $S \subset T$, then $T \in W$.

For a simple game (N, W), each element in W is a subset of N, and it is called a winning coalition. The coalition has enough power to control the decision of the group as a whole under a given decision rule. For example, the majority rule can be expressed as follows:

Example (majority rules). A simple game $G = (N, W)$ is said to be with majority rule, if it is satisfied that for any $S \subset N$, $S \in W$ if and only if $|S| > |N|/2$.

In a simple game, there may be two winning coalitions that are independent each other. However, such a situation is inappropriate in decision rule. Therefore, in this study, we only deal with a winning coalition in an appropriate form. The set of an appropriate winning coalition is defined as "proper simple games" as follows.

Definition 4 (proper simple games). A simple game (N, W) is said to be proper, if for $S, T \in W$, $S \cap T \neq \varnothing$.

By analyzing the actual decision rules, we find that there are decision makers, called "vetoers," who has veto power. In a simple game (N, W), the form of the veto decision maker is decided dependent on the number of decision makers. The term "vetoers" are defined as follows.

Definition 5 (vetoers). A simple game (N, W) is said to be with a vetoer, if there exists $i \in N$ such that for all $S \subset N$, if $S \in W$ then $i \in S$. In this case, DM i is called a vetoer.

Additionally, the game also has decision makers who can form a winning coalition by only one person among the vetoes. Such a decision maker is called "dictator," and it is defined as follows.

Definition 6 (dictators). A simple game (N, W) is said to be a dictator, if there exists $i \in N$ such that for all $S \subset N$, $S \in W$ if and only if $i \in S$. In this case, DM i is called a dictator.

Based on the above definitions, we can define the "committees" as follows. "Committees" is formed by "meetings" and "proper simple games" (Peleg (1984)). The proposal submitted to the committee is called "alternatives." When alternative a is supported by winning coalition S, alternative a has the power to prevent alternate b from being chosen. Such process is expressed as "a dominate b." Using this concept of "dominance relation," we can define "core of committee"

which is a set of alternatives that are likely to be selected by the committee (Inohara (2007)). "Core of committee" is the set of all alternatives that are not dominated by any other alternatives.

> **Definition 7** (committees). A committee $C_R(W) = (N, A, R, W)$ consists of a meeting $C_R = (N, A, R)$ and a proper simple game (N, W).

> **Definition 8** (dominance relation on alternatives). Consider a committee $C_R(W)$, where $C= (N, A)$. For $x, y \in A$ such that $x \neq y$, alternative x is said to dominate alternative y, if there exists $S \in W$ such that for all $i \in S$, xR_iy. x Dom y and $\neg$ (x Dom y) denote that alternative x dominates alternative y, and that alternative x does not dominate alternative y, respectively.

> **Definition 9** (core of committee). Consider a committee $C_R(W)$, where $C= (N, A)$. The core of a committee $C_R(W)$, denoted by $Core(C_R(W))$, is the set of all alternatives that are not dominated by any other alternatives, that is,

$$Core(C_R(W)) = \{ x \in A \mid \forall y \neq x, \neg [y \text{ Dom } x]\}$$

From the above definitions, we can identify the proposal that can be chosen by a committee. The choice of proposal depends on the decision rule adopted by the committee. Decision makers prefer decision rules that determine the desirable alternatives. Therefore, we can assume "a conference to determine decision rule for the main conference that is held to make a decision." By analyzing such meta-situations, we can derive the preference of each decision maker for each decision rule. To express the meta-situation to further extent, the following definitions (Inohara (2007)) are added. The basic structure is same as the definition of groups, meetings, and committees. However, note that the content to be decided is a decision rule rather than a proposal.

> **Definition 10** (meta-groups). Consider a group $C = (N, A)$. The meta-group $\underline{C}$ of the group C is the pair (N, A_N) of the set N of all DMs and the set A_N of all proper simple games on N.

Definition 11 (meta-meetings). Consider a meeting C_R, where $C = (N, A)$. The meta-meeting $\underline{C_R}$ of the meeting C_R is the 3-tuple (N, A_N, R_{CR}), where (N, A_N) is the meta-group $\underline{C}$ of the group $C = (N, A)$, and R_{CR} is a list $(R_{CR,i})_{i \in N}$ of preferences $R_{CR,i}$ of each DM $i \in N$ on A_N.

Definition 12 (meta-committees). Consider a meeting C_R, where $C = (N, A)$, and a proper simple game $W \in A_N$ on N. A meta-committee of the meeting C_R with W, denoted by $\underline{C_R}(W)$, is a 4-tuple $(N, A_N, R_{CR}. W)$, where (N, A_N, R_{CR}) is the meta-meeting $\underline{C_R}$ of the meeting C_R.

3.3. Degree of Satisfaction

Since all decisions are made in response to various levels of problems, it seems impossible to evaluate decision made by each decision maker, based on the unified criterion. Many conferences such as Congresses and meetings in local community deal with both serious and trivial problems. Therefore, the evaluation of the decision depends on the content of the alternative plan, the decision maker's interests, and the political position of the decision maker, etc. It is possible that decision makers do not clarify their preferences and thereby evaluation on the decision is indeed ambiguous and uncertain.

We focus on the preference for decision rules that can be adopted at the conference, rather than the alternatives itself to be decided. Decision rules are usually prearranged in an organization, and the right to decide the decision rule is left to the organizer of the meeting. It is worth noting that preferences for alternatives affect preferences for decision rules adopted at the conference. Decision makers tend to choose decision rules that can realize the alternatives they desire. In other words, the decision maker has preferences for each decision rule, depending on preferences for each alternative. Expressing in another way, policy issues are linked to political issues. Therefore, by grasping the decision maker's preference for the decision rule, it is possible to indirectly clarify the preference for the alternatives.

Each decision maker's preference for each decision rule is called "degree of satisfaction," and "degree of satisfaction" is acquired by deciding alternatives. Even if the seriousness of the agenda varies in practice, all issues are of equal importance. By measuring preferences for decision-making rules, it is possible to avoid ambiguity and uncertainty of preference for alternatives and to introduce unified evaluation criteria in the organization.

"Degree of satisfaction" is derived depending on the "core" of each decision rule in the "meta-committee." We first analyze the "meta-committee" in the conference using the "simple games" framework. We assign points to each decision maker's preference for each decision rule according to the content and number of "core" of the decision rule. To identify each decision makers' preferences in the "meta-committee," Inohara (2007) presents the following method.

Consider a meeting C_R, where $C = (N, A)$, and a DM $i \in N$. For $x \in A$, $\text{rank}_i(x)$ denotes the rank of the alternative x within the preference of DM i, that is,

$$\text{rank}_i(x) = |\{y \in A \mid yP_ix\}| + 1.$$

Note that for $i \in N$ and $x \in A$, $1 \leq \text{rank}_i(x) \leq |A|$. Moreover, let point be a strictly decreasing function from the set $\{1, 2, \ldots, |A|\}$ to the set $\mathbb{R}$ of all real numbers. Then, the point that is assigned to an alternatives $x \in A$ within the preferences R_i of DM i is $\text{point}(\text{rank}_i(x))$.

Moreover, in the case that $Core(C_R(W)) = \varnothing$, "average of the points" that are assigned to the alternatives in the core of a committee $C_R(W)$ that is derived from the meeting C_R by a proper simple game W is

$$\frac{\sum_{x \in Core(C_R(W))} point(rank_i(x))}{|Core(C_R(W))|}$$

denoted by av.pt.$_i(C_R(W))$. Adopting a point function such that $\text{point}(1) = 4$, $\text{point}(2) = 2$, and $\text{point}(3) = 1$, we have av.pt.$_i(C_R(W))$ for each $W \in A_N$ and $i \in N$.

In this study, we use this method to derive "degree of satisfaction" for each decision rule. The detailed procedure of deriving the "degree of satisfaction" is shown below. The point is given 0 if the core is the empty set. By contrast, if no alternative can be chosen, nothing can be decided.

1) Set the point function and give point each alternative according to the preference of each decision maker; if $Core(C_R(W)) = \varnothing$, then av.pt.$_i(C_R(W)) = 0$.

2) Confirm alternatives and their numbers included in the core derived from each decision rule.

3) Based on the average point calculation formula, derive the point of each decision maker in the "meta-committees." In this study, we regard this score as "degree of satisfaction" for each decision rule.

4) Preference for each decision rule of each decision-maker is determined.

3.4. Degree of Dissatisfaction

We derive "degree of dissatisfaction" by comparing "degree of satisfaction" in "meta-committees." Naturally, if the desired plan is different, the desired decision rules are different as well. If decision rules that are undesirable for decision maker are adopted, he or she will be dissatisfied. Generally speaking, the satisfaction and dissatisfaction arise primarily from the inside of individuals, others are not directly involved in their emotions. Therefore, personal satisfaction is essentially absolute and cannot be compared with a satisfaction of others. However, in the present study, the satisfaction of the decision maker and the dissatisfaction of other decision makers are treated as two sides of the same. The gain of a decision maker is the loss of other decision makers. This concept is similar to the limited resource allocation problem. Based on this concept, we compare "degree of satisfaction" of each decision maker.

The method of deriving "degree of dissatisfaction" is extremely simple. For each decision rule, the decision-makers with the highest "degree of satisfaction" are compared with the "degree of satisfaction" of the other decision-makers, and the difference is regarded as "degree of

dissatisfaction." It should be noted that "degree of dissatisfaction" does not arise from the alternative itself, but from decision rules. This "degree of dissatisfaction" corresponds to "external costs" proposed by Buchanan and Tullock (1962). Let us recall that the cause of "external costs" in their theory is decision-making. A series of methods of deriving "degree of satisfaction" and "degree of dissatisfaction" from the meta-situation of decision-making are simple and consistent with Buchanan and Tullocks' theory.

Through the above operation, after "degree of dissatisfaction" of each decision maker is clarified, by summing up those values, "degree of dissatisfaction degree" at that conference can be derived. The dissatisfaction arising during the process of determining an alternative that is undesirable for a decision maker is not correlated with the dissatisfaction of other decision makers. On the other hand, the relationships with their dissatisfaction and the decisions in the conference exist equally. The relationship between the decision at the conference and the dissatisfaction arising from the decision has a network hub structure centered on the decision. Based on the equality of the relationship with the decision, we can simply sum up the "degree of dissatisfaction" of each decision maker, and thereby it becomes possible to measure the degree of conflict between the person who supports the decided alternative and the opponent. It is also possible to compare the degree of conflict between conferences and to clarify the degree of conflict in the organizations hosting a series of conferences.

3.5. Cost of Decision-making

Based on the correspondence between "degree of dissatisfaction" derived in the previous section and "external costs" presented by Buchanan and Tullock (1962), we propose a simple method to derive "decision cost" in the conference. In the "meta-committees," we examine the value of the highest "degree of satisfaction" for decision rules of which core is uniquely determined. The value is called "decision-making costs" in the conference.

This operation is based on the following theory. We need a reasonable cost to achieve satisfaction in decision-making. Decision makers pay some cost so that they can be satisfied in

devising or supporting alternatives. Some examples include information gathering, analysis, examination based on expertise, adjustment of interests, drafting, persuasion, negotiation, etc. These expenses are not always self-owned and may be outsourced, but it is obvious that there is a separate expense in that case. The reason why rational persons incur costs is to gain profit from the decision. From this point, Coase (1938) argues as follows:

> "Costs will only be covered if he chooses, out of the various courses of action which seem open to him, that one which maximizes his profits. To cover costs and to maximize profits are essentially two ways of expressing the same phenomenon" (p. 123).

Briefly, if "degree of satisfaction" is not comparable to the cost required for decision-making, devising or supporting an alternative will generate a loss. If an alternative that is undesirable for a decision maker is determined, he or she will be dissatisfied with the decision, and at the same time, the cost spent on the alternative will be wasted. In other words, "degree of dissatisfaction" introduced in the present model is a concept that measures losses that must be accepted with decisions and losses arising from the inability to recover costs. Based on this concept, we call the highest "degree of satisfaction" value as the "decision-making costs" of the conference. If the decision maker is reasonable, the cost dropped for the decision, and the profit arising from the decision must be comparable. Meanwhile, decision makers who do not benefit from the decision feel dissatisfied with the difference between the "decision-making costs" of the conference and their "degree of satisfaction." The sum of the dissatisfaction is aggregated as "external costs" at the conference.

3.6. Permission and Consensus Building

In this section, we present a framework to model situations where consensus is achieved at the conference. In the modern democratic regime, decisions based on majority rule are regarded as the consensus of the people. Although it is certainly speedy and rational, as stated above, there

is uncomfortable feeling from the viewpoint of the integration of the people's will. While our daily life is filled with a lot of conferences, we should note that we are not always making decisions based on majority rule. There are also many cases that decision makers reach consensus in the conference. In our society, we do not rely on majority voting for all decision-makings, but sometimes we achieve consensus through communication (Habermas (1985)). Modelling the situation where consensus is reached at the conference, we formulate institutionalization of communication-based consensus as a new decision rule and argue that it is a kind of social technology.

In this study, by applying the concept of "permission" to the preference in the "meta-committees," it is shown that a consensus can be achieved at the meta-level. Inohara (2011) devises a way to express the situation where consensus is reached at the conference as "consensus" and "consensus building." Specifically, it is a method of observing the change of the "core" of the conference by introducing the flexibility of the decision maker's preference, using the concept of "permission" by Yamazaki et al. (2000). In this study, we apply this method to the preference of each decision maker in "meta-committees." By using the concept of "permission," it expresses that each decision maker's preference becomes flexible. As a consequence, it shows that "consensus" is achieved by changing "core" in "meta-committees." For ease, "consensus building" is defined to the case where the core is uniquely determined under "unanimity" rule ($W = \{N\}$) after setting "permission." To express "permission," we introduce the following framework proposed by Yamazaki et al. (2000).

Definition 13 (permission). Permission P is a set of acceptable alternatives, that is, $P=(P_i)_{i \in N}$ of permission of DMs i for each GMs $i \in N$, where it is satisfied that $P_i \subset L(A)$ and $R_i \in P_i$ for any $i \in N$. (that is, $R_i \in P_i \subset L(A)$).

Let, for any $R \in L(A)$, *maxR* denotes the most preferred alternatives in *A* in terms of *R*. Let, moreover, for any $S \in W$ and any $a \in A$, *Sa* denotes the set of all DMs in *S* who have in their permission an opinion in terms of which the most preferred alternatives in *A* is *a*, that is, $Sa = \{i \in S, \mid \exists R \in P_i, maxR = a\}$.

Definition 14 (committees with permission) A committee $C = (N, W, A, R)$ with permission *P*, denoted by *C(P)*, is a pair of a committee *C* and a list *P*.

Definition 15 (permission games) The permission game $G_{C(P)}$ is the pair $(N, W_{C(P)})$, where
$$W_{C(P)} = \{S \in W \mid \exists a \in A, Sa \in W\}$$
NOTE: It has been proved by Yamazaki et al. (2000) that the permission game is a simple game.

$W_{C(P)}$ is the set of all winning coalitions. Alternatives must be permitted by a winning coalition in $W_{C(P)}$ in order to be the final choice, and such alternatives constitute the set $A_{C(P)}$ of all permissible alternatives, that is, $A_{C(P)} = \{a \in A \mid \exists S \in W, Sa \in W\}$.

In this study, we apply this framework to the "meta-committees," that is, "permission games" is addressed in the "meta-committees". Before introducing "permission," similar to the "meta-committees", "degree of satisfaction" is derived with "point function" and "average of the point." The method of deriving "degree of dissatisfaction" is analog to "decision-making costs." The feature of "permission game" is to introduce a special point function within the range of "permission." Let us turn to the" permission point function" again. "Permission point function" proposed by Inohara (2007) is the point function with adjustment. For example, in the point function, it assigns 4 points to 1st place, 2 points to 2nd place, and 1 point to 3rd place. At the same time, "permission point function" raises the point given to the preference included in the "permission" range except for the preference of the 1st place. Consequently, it assigns 4 points to

1st place, 3 points to 2nd place and 2 points to 3rd place. This method of bulking represents acceptance of alternatives other than the alternatives most preferred by decision makers. Note that this adjustment has to be carried out so that the original preference order is maintained.

4. Analysis and Results

4.1. Modelling

In this chapter, we analyze simple Congress N by using the framework and method stated in the previous chapter. First of all, let us assume that political parties are strongly constrained by party discipline, like Japan, and let the political party be one decision maker. Under this assumption, it is possible to simplify the subject of decision-making in Congress. If we regard each party as one decision maker, we can perceive that the conference is being conducted by political parties. Let us call the conference "party conference." Indeed, the function of gathering people with close preferences under one flag is one of the main functions of the party. Particularly in Congress adopting the parliamentary cabinet system, it is well known that party disciplines tend to be strong. In Japanese Diet (Parliament), such tendency is quite strong. Each legislator is not a completely independent entity and takes actions by policies of his or her political party. Congress with strong party discipline is less common, but it is convenient to simplify the process of congressional decision-making situation. Analysis of Congress with weak party discipline like the United States Congress is too complicated for us at this stage.

Next, by analyzing the pattern of "winning coalition" in "party conference," we grasp the pattern of "decision rule" that is substantially adopted in Congress N. Let us assume that Congress N consists of one ruling party and two opposition parties ($N = \{\text{DM1, DM2, DM3}\}$). DM1 is a ruling party with a majority of seats and has the status of "dictator." Inevitably, "party conference" adopts the "dictatorial" rule that DM1 can demonstrate strong determination power. Therefore, in

Congress where the party members are strongly restrained, it can be found that the "majority" rule is converted into "dictatorial" rule.

However, even in "party conference," the adoption of "majority" rule and "unanimity" rule is not completely excluded. Because DM1 is tolerant of the existence of a winning coalition that includes itself. DM1 has the status of "dictator," rather than always makes decisions with only one person. If there is an opposition party who agrees with their ideas, it is possible that "majority" rule or "unanimity" rule will be adopted. Indeed, in the Upper House of Japan, the "unanimity" rule is adopted as a rule for the committee to submit a draft legislation. As a result, here we have three distinct patterns of decision rule: (i)"dictatorial" rule; (ii)"majority" rule; and (iii)"unanimity" rule. We will analyze these three decision rules.

4.2. Analysis and Results

In this section, we analyze Congress N = {DM1, DM2, DM3} assumed in the previous section. First, elements necessary for analysis are determined as follows.

N = {DM1, DM2, DM3},

R: DMs' preference list on A,

Set of alternatives A = {a, b, c},

W: set of proper simple games,

meeting C_R = {N, A, R},

committee $C_R(W)$ = (N, A, R, W),

meta-meeting $\underline{C_R}$ = (N, A_N, R_{CR}),

meta-committee $\underline{C_R}(W)$ = (N, A_N, R_{CR}, W),

A_N = {"dictatorial" rule, "majority" rule, "unanimity" rule},

R_{CR}: DMs' preference list on A_N,

Core (*CR* (*W*)): Core of committee (alternatives not controlled by any alternative).

Let $N = \{DM1, DM2, DM3\}$, then we have seven possible non-empty coalitions, that is, $\{DM1\}, \{DM2\}, \{DM3\}, \{DM1, DM2\}, \{DM2, DM3\}, \{DM3, DM1\}$, and $\{DM1, DM2, DM3\}$. They are denoted by 1, 2, 3, 12, 23, 31, and 123, respectively. We have 11 proper simple games (N, W) with N as follows: $W_a = \{1, 12, 31, 123\}$, $W_b = \{12,123\}$, $W_c = \{31,123\}$, $W_d = \{12,31,123\}$, $W_e = \{12,31,23,123\}$, $W_f = \{123\}$, $W_g = \{2,12,23,123\}$, $W_h = \{23, 123\}$, $W_i = \{12, 23, 123\}$, $W_j = \{3, 23, 31, 123\}$, and $W_k = \{23, 31, 123\}$. As already mentioned in the previous section, we only need to analyze W_a ("dictatorial" rule), W_e ("majority" rule), and W_f ("unanimity" rule).

The preference list of each decision maker is determined and is shown by Table 1.

Table 1. Preference List of Decision Makers

Rank	DM1	DM2	DM3
1st	*a*	*c*	*c*
2nd	*b*	*b*	*b*
3rd	*c*	*a*	*a*

We introduce "point function" and "average of the points" devised by Inohara (2007). Details are as shown in Table 2.

Table 2. "Average of the Points" and "Point Function"

Average of the Points	Point Function	
	Rank	Point
$\dfrac{\sum_{x \in Core(C_R(W))} point(rank_i(x))}{|Core(C_R(W))|}$	1st	4
	2nd	2
	3rd	1

NOTE: if $Core\ (C_R\ (W)) = \varnothing$, then av.pt.$_i$ $(C_R\ (W)) = 0$.

Using "the average of the point" and "the point function," we can derive "degree of satisfaction" of each decision maker. The results are shown in Table 3. The preference order of each decision maker for each decision rule is shown in the following table 4.

Table 3. "Degree of Satisfaction" of Decision Makers

Decision Rule	Core of Committee	DM1's av.pt.$_i$ $(C_R(W))$	DM2's av.pt.$_i$ $(C_R(W))$	DM3's av.pt.$_i$ $(C_R(W))$
Dictatorial	$\{a\}$	4	1	1
Majority	$\{c\}$	1	4	4
Unanimity	$\{a, b, c\}$	2.3	2.3	2.3

Table 4. Preference Order for Each Decision Rule

Rank	DM1	DM2	DM3
1st	Dictatorial	Majority	Majority
2nd	Unanimity	Unanimity	Unanimity
3rd	Majority	Dictatorial	Dictatorial

Furthermore, we derive "degree of satisfaction" and "degree of dissatisfaction "of each decision maker. The "degree of satisfaction" corresponds to "decision-making costs," "degree of dissatisfaction" corresponds to "external costs." The results are as shown in Table 5. Under the "dictatorial" rule, we can observe that "external costs" exceeds "decision-making costs."

Table 5. "Decision-making Costs" and "External Costs"

Decision Rule	Core of Committee	DM1's av.pt.$_i$ ($C_R(W)$)	DM2's av.pt.$_i$ ($C_R(W)$)	DM3's av.pt.$_i$ ($C_R(W)$)	Decision-making Costs	External Costs
Dictatorial	$\{a\}$	4	1	1	4	6
Majority	$\{c\}$	1	4	4	4	3
Unanimity	$\{a, b, c\}$	2.3	2.3	2.3	-*	-*

＊ If the core is not uniquely determined, there is no satisfying person, so no cost will occur.

Next, let us consider the case where "permission" is introduced in "meta-committees". The range of permission of each decision maker has various patterns. In this paper, as an example, "permission" is set in the form Table 6 (Case 1) and Table 7 (Case 2).

Table 6. "Permission" Range of Decision Makers (Case 1)

Rank	DM1	DM2	DM3
1st	a	c	c
2nd	b	b	b
3rd	c	a	a

Table 7. "Permission" Range of Decision Makers (Case 2)

Rank	DM1	DM2	DM3
1st	a	c	c
2nd	b	b	b
3rd	c	a	a

To clarify "degree of satisfaction" under "permission" situation, we use "permission point function" instead of the "point function." Details are shown in Table 8.

Table 8. "Permission Point Function"

Rank	Permission Point Function
1st	4
2nd	3*
3rd	2*

* Within the permissible range, while maintaining the original preference order, it raises points for alternative plans of second or lower.

As a result, "core of committee" is uniquely determined under the unanimity rule. The procedure in detail is illustrated by Table 9 (case 1) and Table 10 (case 2).

Table 9. Results of Consensus Building (case 1)

Decision Rule	Core of Committee	DM1's av.pt.$_i$ ($C_R(W)$)	DM2's av.pt.$_i$ ($C_R(W)$)	DM3's av.pt.$_i$ ($C_R(W)$)	Decision-making Costs	External Costs
Dictatorial	$\{a\}$	4	1	1	4	6
Majority	$\{c\}$	1	4	4	4	3
Unanimity	$\{a, b, c\}$	2.3	2.3	2.3	-*	-*
Consensus Building**	$\{b\}$	3	3	3	3	0

* If the core is not uniquely determined, there is no satisfying person, and thereby no decision cost will occur.

** "Consensus Building" is defined to a case where the core is uniquely determined under "unanimity" rule ($W = \{N\}$) after setting "permission."

Table 10. Results of Consensus Building (case 2)

Decision Rule	Core of Committee	DM1's av.pt.$_i$ ($C_R(W)$)	DM2's av.pt.$_i$ ($C_R(W)$)	DM3's av.pt.$_i$ ($C_R(W)$)	Decision-making Costs	External Costs
Dictatorial	{a}	4	1	1	4	6
Majority	{c}	1	4	4	4	3
Unanimity	{a, b, c}	2.3	2.3	2.3	-*	-*
Consensus Building**	{a}	4	2	2	4	4

* If the core is not uniquely determined, there is no satisfying person, and thereby no decision cost will occur.

** "Consensus Building" is defined to a case where the core is uniquely determined under "unanimity" rule ($W = \{N\}$) after setting "permission."

By applying the consensus building rule, it is found that the core is uniquely determined and "external cost" has been drastically reduced (case 1). However, we also find that consensus building rules do not always give optimal results (case 2).

5. Discussion

So far, we have proposed a method of measuring "cost of decision-making" that can be applied to actual conferences, which can be regarded as the novelty of the present study. The method is based on the important concept called "cost of decision-making" (Buchanan and Tullock (1962)). However, so far little attention has been paid to measurement of the cost, and our study shed light on this problem. It is needed to quantify the cost so that conflicts and consensus in the conference can be measured. The feature of our method was to derive "cost of decision-making"

from the meta-situation of decision-making, rather than from proposals. Making use of this method, it is possible to avoid the complexity of setting each preference of decision makers for each proposal. We can avoid uncertainty depending on the situation. Our new approach using the game theoretical framework may become a unified way to evaluate the degree of consensus at all decision-making organizations, including Congress and public committees.

However, at present, the investigation is not yet sufficient. We have discussed the consistency of the theory to derive "cost of decision-making," but the way to derive the equilibrium for minimization of the cost is not studied in the present model. Furthermore, since our method is still in an elementary stage, the validity of the method remains questionable in terms of its application to reality. Detailed verification regarding applicability to reality is required. Also, this study only considers a very simple situation. Realistic conferences will usually be carried out by more participants. To apply to complicated conferences, it is needed to improve our models and methods.

By analyzing Congress with strong party discipline, it is possible to simplify the decision-making situation. We confirm a high-level conference called "party conference" within Congress. Generally speaking, strong party discipline strengthens the consistency of political parties' policies. Citizens can easily grasp the position of each party and the conflict point between political parties. In countries that adopt the parliamentary cabinet system, the party discipline brings about the stability of the administration and the smooth legislative process.

However, in "party conference," it should be noted that "majority rule" which is considered as a uniform and basic rule of Congress is converted to another rule. Differences in the rule environment may affect the decision-making situation. Based on the results of the analysis, we clarified that there are three decision rules in "party conference," "dictatorial," "majority," and "unanimity." If the ruling party who won the majority of seats rely on the "dictatorial" rule, "majority" rule and "unanimity" rule will be neglected. Indeed, under the strong party discipline, drafting the bill will be carried out outside Congress (e.g., the expert committee within the ruling party), and the bill is rarely amended after being submitted to Congress and passed. Under the

parliamentary cabinet system, although there are strong criticisms against such a situation, the situation occurs from the rational behavior of Congress. We are not in a position to judge its appropriateness. At least, however, if the citizen's integration is hindered by the rational behavior of Congress, it will result in tragedy for the country that adopts democracy. In this sense, policies for politics become important, and policies that prevent "the tragedy of the commons" are required.

As a finding of the analysis in this study, we show that it is possible to make decisions using methods other than "dictatorial" rule, "majority" rule, and "unanimity" rule. It is a method of preparing multiple alternatives and setting "permission" in the preference order. Consequently, "consensus" may be achieved. Let us rule this process and call it "consensus building." In the present situation, there are some cases where consensus is achieved by the efforts of each decision maker. However, since a lot of effort is required to achieve the consensus, there are not many cases of consensus. Especially when the "dictatorial" rule is adopted, seeking consensus is a waste of time for the ruling party. However, in democracy, there are many problems where consensus is indispensable for policy implementation. For example, it is a problem related to constitutional amendment and rules on legislation. If the "consensus building" rule is introduced into a conference as a new decision rule, it can be a tool to improve the problem of consensus in democracy. At least, we can expect the effect of converting "power game (i.e., power struggle)" based on the number of seats into "cooperative game (i.e., bargaining)". The merit of this rule can be summarized as that the consensus can be systematically derived while the preference order of each decision maker is sustained. When "consensus building" rules are applied in society, decision makers' behavior and necessary qualities will also change. New decision rules in Congress may change the behavior of people and the concept of democracy.

However, from the results of the analysis, it can also be found that the "consensus building" rule does not always lead to optimum results. In using the "consensus building" rule, when each decision maker does not expand the range of "permission," it ends with meaningless efforts. Some compromise between decision makers is indispensable to "consensus building" rule be meaningful. Alternatively, each decision maker must consider the preferences of others in drafting alternatives.

On the other hand, it should also be noted that the objective of the decision is not only to reach an agreement but also to solve the problem. It is meaningless to decide a proposal without effect. Needless to say, we cannot deal with all social problems using "consensus building" rules. It is advisable to properly deal with problems without unduly relying on "consensus building" rules. In order to apply "consensus building" rules to Congress, it is needed to overcome high-level problems. When changing decision rules in Congress, it is needed to fully consider the influence on the Constitution and the governance system. Therefore, at this stage, it only shows that the "consensus building" rule could be an option to improve the operation of democracy. For the "consensus building" rule to be recognized as an effective policy to realize good politics, more interdisciplinary and sophisticated investigation will be required.

6. Conclusion

In this study, we present an approach to measure "cost of decision-making" (Buchanan and Tullock (1962)) and the degree of consensus in the conference. We first model and analyze the conference with the game theoretical framework called "simple games" (Inohara (2007)). Subsequently, we consider the meta-situation, that is "a conference to determine decision rule for the main conference that is held to make a decision," and derived the "degree of satisfaction" and "degree of dissatisfaction" of each decision-maker for each decision rule. Using these indicators, we derived "decision-making costs" and "external costs" in the conference. Finally, we proposed a method to measure the degree of consensus at the conference by comparing these costs. Also, we provide a method to measure the effect of "consensus building" by using the concept expressing the flexibility of preference called "permission" proposed by Yamazaki et al. (2000).

The novelty of this study is that we propose a method of measuring "cost of decision-making" that can be applied to actual conferences. Our method is based on the important theory of "cost of decision-making." The method of measuring the cost has not been sufficiently studied, and our model shed light on this problem. We proposed a method to measure "cost of decision-

making" in the conference and applied it to clarify the nature of conflicts and consensus. The feature of our method was to derive "cost of decision-making" from the meta-situation of decision-making rather than from proposals. Making use of this method, we find it is possible to set each preference of decision makers for each proposal in the relatively simple way, and further this method can also avoid uncertainty depending on the fluid situation.

This study has two implications: (i) Our new approach using the game theoretical framework may become a unified way to evaluate the degree of consensus at all decision-making organizations, including Congress and other public committees. (ii) The result of our analysis suggests the possibility of applying "consensus building" (Inohara (2011)) rule to the conference in Congress. In any case, for our method to be recognized as an effective policy to realize good politics, more interdisciplinary and sophisticated investigations will be required.

Acknowledgments

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